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Part II. Case Study of the Effect of the Dimensionless Groups and Comparison with Experimental Data

A case study is made to determine the dependence of the deep bed filtration rate (expressed in terms of the fraction of suspended particles impacted) on the eight dimensionless parameters, which are found to be relevant in the filtration process, based on the trajectory calculation method developed in Part I of this series. In addition, comparisons between results based on the theoretical model of this work and available experimental data are made. Comparisons are also made with some of FitzPatrick's theoretical results. The results of this study demonstrate clearly the complex and interactive nature of the relation between the various parameters and the efficiency of filtration. Accordingly, the conventional format of correlating experimental data, which equates the filter coefficient with a product of the pertinent dimensionless groups, each raised to an empirical exponent, will not be adequate in providing a generalized correlation of experimental filtration data.

SCOPE

The use of dimensional analysis for correlating deep bed filtration data was made rather recently by Ison and Ives (1969) in an attempt to establish the dependence of the initial filter coefficient on a few of the dimensionless parameters pertinent to deep bed filtration. More recently FitzPatrick (1972) proposed a theoretical model of deep bed filtration, based on which he calculated the dependence of the initial filter coefficient on some of the dimensionless groups appearing in his mathematical formulation.

In this part of this series, the theoretical model of deep bed filtration and the method for its solution developed in Part I are used to make a more complete study of the dependence of the fraction impacted on the dimensionless parameters appearing in the mathematical formulation. Apart from the fact that the porous media model used in the present study (P-T-T model) is more realistic than models used by previous investigators, the present study is also more complete since it includes the retardation effect on the molecular dispersion force.

This study concludes with comparisons between theoretical results and experimental data, as well as comparisons with some of FitzPatrick's theoretical results.

CONCLUSIONS AND SIGNIFICANCE

The dependence of the fraction (of suspended particles) impacted, η_0, on each of the dimensionless groups appearing in the trajectory equation is determined in a case study. It is found that when the molecular dispersion force dominates the double layer interaction force at all separations the effect of the latter is negligible, but a sharp drop of η_0 (or of the initial filter coefficient, λ_0) in excess of three orders of magnitude is observed for any change of the parameters that leads to a situation in which a repulsive double layer interaction force becomes sufficiently larger in magnitude than the attractive molecular dispersion force at separations larger than a critical one. Such situations may take place through a combination of several factors. The dependence of η_0, (or λ_0) on the various dimensionless groups is such that the quantity $\lambda_0 \langle d_g \rangle$ cannot be expressed as a product of the powers of the various dimensionless groups over extended ranges of these groups, and the simple product expression suggested by Ison and Ives (1969) has only limited validity over small ranges of the dimensionless groups. For studies over wide ranges of the group values, the designer has to resort to additional experimentation or to models such as the one developed in the present work.

Experimental results obtained by Ison and Ives (1969) and Craft (1969) are compared with the theoretical calculations based on the model developed in this work, and in general, they are found in good agreement. In addition, theoretical values of the filter coefficient obtained by FitzPatrick (1972) corresponding to experimental data obtained by FitzPatrick (1972) and Ison (1967) are compared with theoretical values based on the model of the present work.

CASE STUDY OF THE EFFECT OF THE DIMENSIONLESS GROUPS ON THE FRACTION IMPACTED

A useful result of a theoretical model is that it enables the study of a phenomenon under conditions which are difficult to duplicate in practice. The study of the dependence of any phenomenon on the pertinent dimensionless groups is always useful since one thus obtains information on the dependence of the phenomenon on the various dimensional parameters comprising the dimensionless groups in concise form. As has been demonstrated in Part I of this series, deep bed filtration involves at least 8 such dimensionless groups, and, as it should be expected, the effect due to a change of the value of any one of these groups cannot be isolated from the values of all other groups. This, unfortunately, would render a complete investigation of the effect of the dimensionless groups a very time-consuming undertaking. For this reason, the simpler, but incomplete, approach of a case study was preferred.

A set of pertinent parameters typical of deep bed filtration was selected and given in Table 1. Subsequently, the dependence of η_0 on each one of the dimensionless groups was determined by varying the value of the particular group under consideration and assigning to all other groups their basic values given in Table 1. The results are summarized below.

Effect of the Gravitational Group, $N_G = 2(\rho_p - \rho)\, a_p^2 g/(9\mu v_s)$

The relation between the fraction impacted η_0 and the gravitational group N_G is shown in Figure 1. For small values of N_G, η_0 is a strong increasing function of N_G, but this dependence diminishes as N_G increases. This is so because for small values of N_G the gravitational force exerted on the particle is of comparable magnitude to the hydrodynamic force far from the wall and, therefore, a small change in the value of N_G results in a considerable change on the location of the part of the limiting trajectory far from the immediate vicinity of the wall, whereas for large values of N_G the gravitational force dominates the hydrodynamic force, and, therefore, the part of the limiting trajectory far from the immediate vicinity of the wall is almost parallel to the direction of gravity and changes little even for large increases of the N_G value.

Effect of the Relative Size Group, $N_{RS} = a_p/\langle d_g \rangle$

The relation between η_0 and the relative size group N_{RS} is shown in Figure 2. The η_0 vs. N_{RS} curve passes through a minimum. For a given bed, N_{RS} variations must be interpreted as variations of the particle radius a_p. Since all forces exerted on the particle depend on the particle radius, the shape of the curve in Figure 2 should be considered as the net result of the relative magnitude of the forces for various particle diameter values with the other dimensional parameters adjusted so that the rest of the dimensionless groups remain constant.

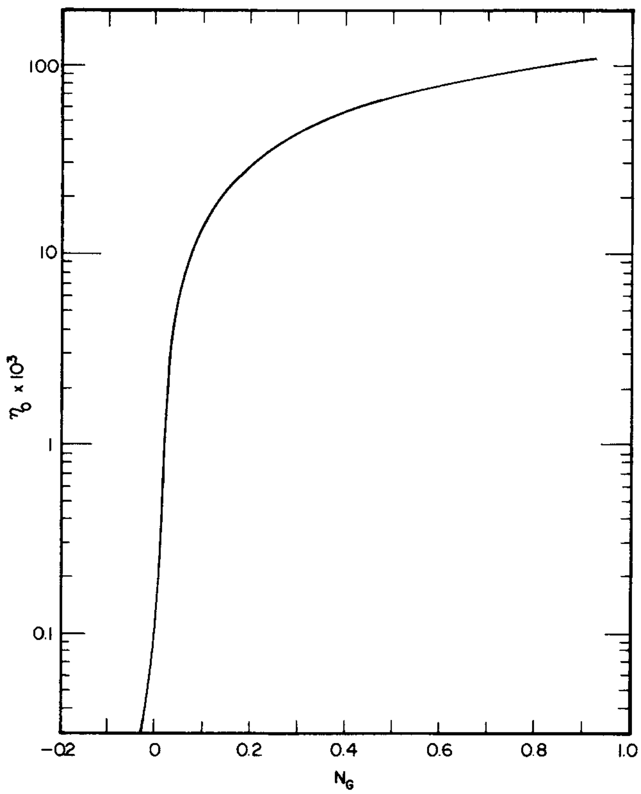

Fig. 1. Theoretical relation between η_0 and N_G, (all other dimensionless groups constant).

TABLE 1. VALUES OF THE PARAMETERS AFFECTING FILTRATION USED AS A BASIS FOR THE STUDY OF THE EFFECT OF THE VARIOUS DIMENSIONLESS GROUPS ON THE FRACTION IMPACTED, AND CORRESPONDING VALUES OF THE DIMENSIONLESS GROUPS

Parameter	Value	Parameter	Value	Group	Value
a^*	0.804	μ, poise	0.008937	N_G	2.2584×10^{-2}
d^*	0.337	ρ, g cm^{-3}	0.99708	N_{RS}	7.003×10^{-3}
$\langle d_g \rangle$, cm	0.0714	v_s, cm s^{-1}	0.1358	N_{E1}	53.095
$\langle d_g^2 \rangle$, cm^2	5.098×10^{-3}	$2a_p$, cm	0.0010	N_{E2}	0.4979
$\langle d_g^3 \rangle$, cm^3	3.640×10^{-4}	ρ_p, g cm^{-3}	1.5	N_{DL}	1400
$\langle d_c \rangle$, cm	0.0241	$\tilde{\epsilon}$	81	N_{L0}	5.828×10^{-5}
$\langle d_c^3 \rangle$, cm^3	1.692×10^{-5}	κ, cm^{-1}	2.8×10^6	N_{Ret}	314.159
N_c, cm^{-2}	178	ψ_{01}, mV	−30	$(N_{Re})_s$	1.082
ϵ_0	0.41	ψ_{02}, mV	−8		
l, cm	0.0686	H, erg	5×10^{-13}		
T, °C	25.0				

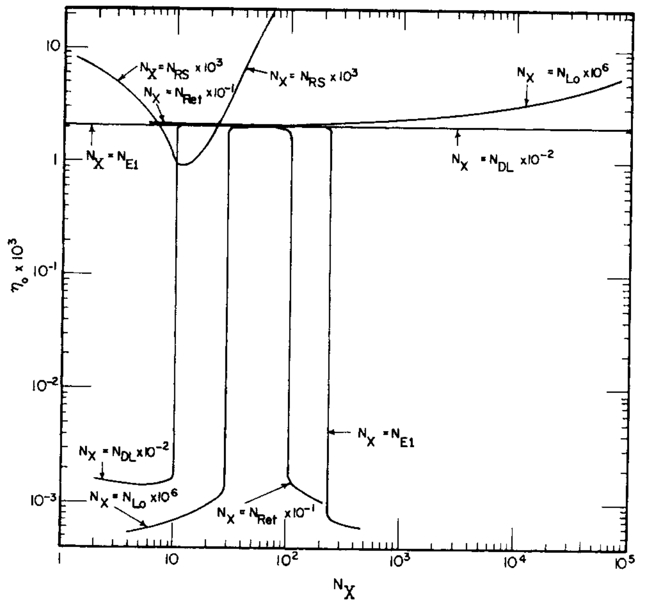

Fig. 2. Theoretical relation between η_0 and N_{RS}, N_{E1}, N_{DL}, N_{Lo}, and N_{Ret} (varying only one group at a time).

Effect of the Groups $N_{E1} = \tilde{\epsilon}\kappa(\psi^2{}_{01} + \psi^2{}_{02})/(12\pi\mu v_s)$, $N_{DL} = \kappa a_p$, $N_{Lo} = H/(9\pi\mu a_p{}^2 v_s)$, and $N_{\text{Ret}} = 2\pi a_p/\lambda_e$

As can be seen in Figure 2, there is a short range around a critical value for each one of the above dimensionless groups (in the present case study, for example, $N_{E1} \cong 230$, $N_{DL} \cong 1000$, $N_{Lo} \cong 2.9 \times 10^{-5}$, $N_{\text{Ret}} \cong 1100$) in which a sharp decrease, in excess of three orders of magnitude, of the value of η_0 is observed. The low η_0 value segment of the curve for each one of these groups corresponds to the case where the repulsive double layer force is strong enough to hinder deposition, that is the case when $F_{Ly}{}^{\text{ret}} + F_{Ey} + F_{Gy} + F_{Dy} > 0$ for $\delta^+ > \tilde{\delta}^+{}_{cr}(\zeta_1{}^+) \cong \delta^+{}_{cr}$ (see Part I of this series). The shape of the low η_0 value segment in each case depends (among other factors) on the expression used for the double layer force F_{Ey}, in this case Equation (27) of Part I, which is based on the Hogg, Healy, and Fuerstenau (1966) model. As discussed in the Supplement,* this model is not valid for very small separations, and therefore the validity of the low η_0 value segments for the groups in consideration is uncertain. Furthermore, if one assumes that in the case when $F_{Ly}{}^{\text{ret}} + F_{Ey} + F_{Gy} + F_{Dy} > 0$ for $\delta^+ > \tilde{\delta}^+{}_{cr}(\zeta_1{}^+)$, no particles exist at the unit cell entrance at positions close to the wall within the potential energy barrier, then the low η_0 value segment for each of these groups should be $\eta_0 = 0$. The high η_0 value segment for each of these groups corresponds to the case when the double layer force is negligible at all separations between suspended particle and grain, that is, $F_{Ly}{}^{\text{ret}} + F_{Ey} < 0$ for $\delta^+ \geqq 0$, whereas the segment of sudden change corresponds to the incipient case. As shown in Figure 2, when the double layer force is negligible, the effect of N_{E1} and N_{DL} on η_0 is also insignificant, but there is considerable dependence on N_{Lo} and N_{Ret}, η_0 being an increasing function of N_{Lo} and a decreasing function of N_{Ret}.

Effect of the Second Electrokinetic Group, $N_{E2} = 2\psi_{01}\psi_{02}/(\psi^2{}_{01} + \psi^2{}_{02})$

Negligible dependence of η_0 on N_{E2} is observed, as it should be expected, since the parameter values used for the calculations (Table 1), are such that the London force dominates the electrokinetic force at all separations. In cases in which $F_{Ly}{}^{\text{ret}} + F_{Ey} + F_{Gy} + F_{Dy} > 0$ for $\delta^+ > \tilde{\delta}^+{}_{cr}(\zeta_1{}^+) \cong \delta^+{}_{cr}$, one expects η_0 to be a monotonically decreasing function of N_{E2}, with all the above mentioned reservations concerning the validity of results obtained based on Equation (27) for very small separations (see Supplement).

Effect of the Superficial Reynolds Number, $(N_{Re})_s = \langle d_g \rangle v_s/\nu$

The superficial Reynolds number $(N_{Re})_s$ does not appear explicitly in the trajectory equation [Equation (38), Part I], but the variables of $A_i{}^+$, $B_i{}^+$, and $D_i{}^+$ depend on $(N_{Re})_i$, and therefore, on $(N_{Re})_s$. However, for creeping flow, $A_i{}^+$, $B_i{}^+$, and $D_i{}^+$ no longer depend on $(N_{Re})_i$, [neither on $(N_{Re})_s$], and under this condition, η_0 does not depend on $(N_{Re})_s$. This, however, does not mean that η_0 is independent of quantities such as $\langle d_g \rangle$, v_s, and ν since they also appear in other dimensionless groups on which η_0 depends. It means that for a series of experiments in which all dimensionless groups N_G, N_{RS}, N_{E1}, N_{E2}, N_{DL}, N_{Lo} and N_{Ret} are constant and $(N_{Re})_s$ varies within the regime of creeping flow then, according to the postulated model, one should observe the same η_0 for all experiments. This was confirmed by actual computation.

On Data Correlations

Ison and Ives (1969) proposed to express the quantity $\lambda_0 \langle d_g \rangle$ as being proportional to each one of the dimensionless groups raised to an empirical exponent and had limited success in fitting Ison's experimental data (1967) into such an expression, which included the gravitational group, the interception group,* and the superficial Reynolds number. However, expressions of this form are only successful when the logarithm of $\lambda_0 \langle d_g \rangle$ (or, for that matter,† the logarithm of η_0) is a linear function of the logarithm of each one of the dimensionless groups considered, and, as it was shown above, this condition is not satisfied over wide ranges of the dimensionless groups. The power law format therefore cannot be considered as a basis for generalized correlation. It seems that one has to resort to the use of models, such as the one developed in the present work, to study deep bed filtration systems under substantially varying conditions.

COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL FILTRATION RESULTS

Comparison with Data by Ison and Ives (1969)

Ison and Ives obtained filtration data using suspensions of kaolinite in tap water and randomly packed beds of ballotini (glass spheres). Five sets out of a total of eight of their data were obtained with beds composed of ballotini with an average diameter of 460μ, and the remaining three with beds composed of ballotini with average diameters of 548μ, 658μ, and 777μ, respectively. For the calculation of the theoretical filter coefficients corresponding to the experimental conditions, the characteristics of the filter beds in terms of the P-T-T model, such as constriction size distribution, etc., were estimated in this work using a procedure outlined in the Supplement. In addition, a number of other parameters, unreported in the original work, had also to be estimated. The values of these quantities are summarized in Table 2. Based on these values, theoretical calculations of η_0 were made and were compared with the Ison and Ives data as shown in Figures 3 and 4. In addition, values of the fraction impacted due to Brownian motion alone, η_{BMo}, were calculated using the method of Pfeffer and Happel (1964) as suggested by Cookson

* See footnote on page 891.

* The interception group is defined as $2a_p/\langle d_g \rangle$.

† Using Equation (7) of Part I, one has $\lambda_0 \cong \frac{\eta_0}{l}$ (for $\lambda_0\, l \ll 1$, which is generally true).

TABLE 2. PARAMETER VALUES USED FOR THE CALCULATION OF THE THEORETICAL RESULTS IN FIGURES 3 AND 4

Parameter \ Run	I	II	III	IV	V	VI	VII	VIII
a°	0.795‡	0.795‡	0.795‡	0.795‡	0.795‡	0.795‡	0.795‡	0.795‡
d°	0.352‡	0.352‡	0.352‡	0.352‡	0.352‡	0.352‡	0.352‡	0.352‡
$<d_g>$, cm	0.0460	0.0460	0.0460	0.0460	0.0460	0.0548	0.0658	0.0777
$<d_g^2> \times 10^3$, cm^2	2.1219‡	2.1219‡	2.1219‡	2.1219‡	2.1219‡	3.0114‡	4.3417‡	6.0540‡
$<d_g^3> \times 10^5$, cm^3	9.8184‡	9.8184‡	9.9184‡	9.8184‡	9.8184‡	16.600‡	28.737‡	47.318‡
$<d_c>$, cm	0.0162‡	0.0162‡	0.0162‡	0.0162‡	0.0162‡	0.0193‡	0.0231‡	0.0273‡
$<d_c^3> \times 10^6$, cm^3	4.8949‡	4.8949‡	4.8949‡	4.8949‡	4.8949‡	8.2758‡	14.327‡	23.590‡
N_c, cm^{-2}	441‡	441‡	441‡	441‡	441‡	311‡	216‡	155‡
ϵ_0	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38
l, cm	0.0436‡	0.0436‡	0.0436‡	0.0436‡	0.0436‡	0.0520‡	0.0624‡	0.0737‡
ϕ_s	1	1	1	1	1	1	1	1
T, °C	20	25	30	16	13	20	20	20
μ, poise	0.010	0.009	0.008	0.011	0.012	0.010	0.010	0.010
ρ, g cm^{-3}	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
v_s, cm s^{-1}	0.159	0.143	0.127	0.175	0.191	0.159	0.159	0.159
ρ_p, g cm^{-3}	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
$\tilde{\epsilon}$	81	81	81	81	81	81	81	81
κ, cm^{-1}	2.8 × 10^6‡	2.8 × 10^6‡	2.8 × 10^6‡	2.8 × 10^6‡	2.8 × 10^6‡	2.8 × 10^6‡	2.8 × 10^6‡	2.8 × 10^6‡
ψ_{01}, mV	−55‡	−55‡	−55‡	−55‡	−55‡	−55‡	−55‡	−55‡
ψ_{02}, mV	−7‡	−7‡	−7‡	−7‡	−7‡	−7‡	−7‡	−7‡
$H \times 10^{13}$, erg	5°	5°	5°	5°	5°	5°	5°	5°

‡ Estimated. ° Estimated in Payatakes (1973).

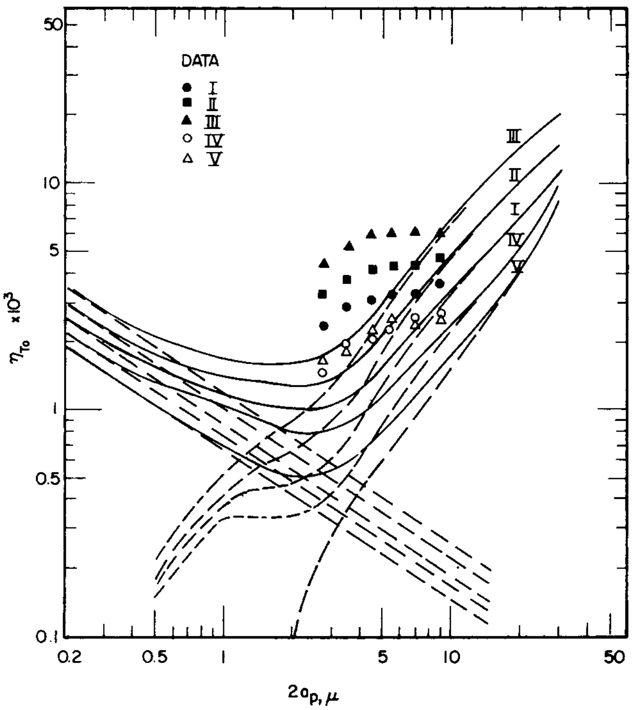

Fig. 3. Comparison between Runs I to V by Ison and Ives and the corresponding calculated values.

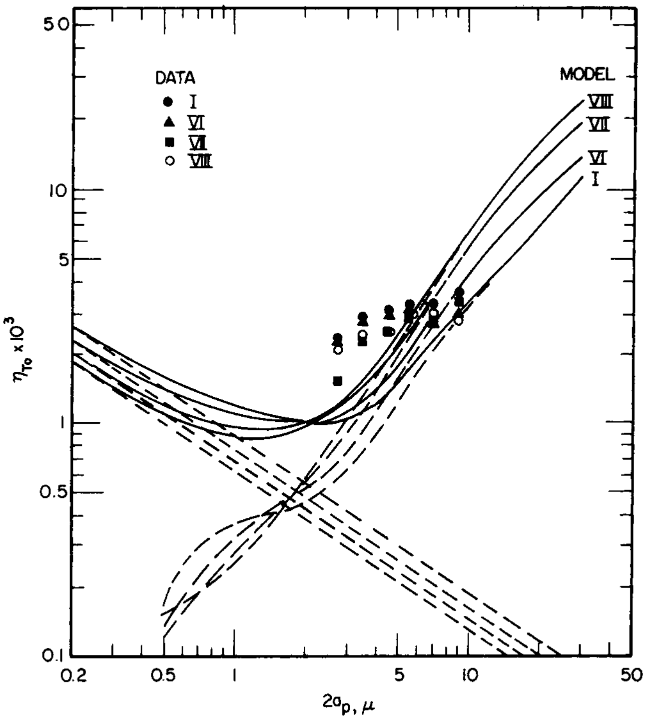

Fig. 4. Comparison between Runs I, VI, VII, and VIII by Ison and Ives, and the corresponding theoretical values.

(1970) and are shown as dotted lines on these figures. If one assumes that the total fraction impacted can be obtained by adding η_{BM0} to the value of the fraction impacted due to non-Brownian forces, as was done by Yao (1968), one has

$$\eta_{T0} = \eta_{BM0} + \eta_0 \tag{1}$$

where η_{T0} is the total fraction impacted, taking into consideration gravitational, hydrodynamic, electrokinetic, London, as well as Brownian forces. The use of Equation (1) can only be justified on the basis that it gives a good agreement to experimental data since it does not have a theoretical basis. The solid lines in Figures 3 and 4 give the value of η_{T0}. As can be seen, the theoretical model is in good qualitative agreement with the experimental data. The quantitative agreement between theoretical and experimental results is good for runs I to V and very good for runs VI to VIII although the theoretical dependence on the suspended particle diameter is somewhat stronger than the experimental one.

While the comparisons are generally good, especially in view of the experimental error bounds reported by Ison (1967), the comparisons are not exact. The various porous media model parameters were estimated as mentioned earlier. Another important factor is the nonsphericity of the suspended particles used in experiments. Unfortunately, the possible effect of this last factor, cannot be assessed with the present model.

Comparison with Data by Craft (1969)

Craft obtained several sets of data using suspensions of vermiculite in tap water and randomly packed sand beds.

TABLE 3. PARAMETER VALUES USED FOR THE CALCULATION OF THE THEORETICAL RESULTS IN FIGURES 5 AND 6

Parameter \ Run		Figure 5			Figure 6	
	C.I	C.II	C.III	F.I	F.II	F.III
a^*	0.804‡	0.804‡	0.804‡	0.795‡	0.795‡	0.795‡
d^*	0.337‡	0.337‡	0.337‡	0.352‡	0.352‡	0.352‡
$<d_g>$, cm	0.0335	0.0714	0.1122	0.0720	0.0720	0.0720
$<d_g^2> \times 10^3$, cm^2	1.1222‡	5.0980‡	12.589‡	5.1983‡	5.1983‡	5.1983‡
$<d_g^3> \times 10^5$, cm^3	3.7595‡	36.399‡	141.25‡	37.650‡	37.650‡	37.650‡
$<d_c>$, cm	0.0113‡	0.0241‡	0.0378‡	0.0253‡	0.0253‡	0.0253‡
$<d_c^3> \times 10^6$, cm^3	1.7473‡	16.917‡	65.646‡	18.770‡	18.770‡	18.770‡
N_c, cm^{-2}	809‡	178‡	72‡	180‡	180‡	180‡
ϵ_0	0.41‡	0.41‡	0.41‡	0.39	0.39	0.39
l, cm	0.0322‡	0.0686‡	0.1078‡	0.0683‡	0.0683‡	0.0683‡
ϕ_s	0.73‡	0.73‡	0.73‡	1.0	1.0	1.0
T, °C	25.0	25.0	25.0	20.0	20.0	20.0
μ, poise	0.008937	0.008937	0.008937	0.01005	0.01005	0.01005
ρ, g cm^{-3}	0.99708	0.99708	0.99708	1.000	1.000	1.000
v_s, cm s^{-1}	0.1358	0.1358	0.1358	Varies	Varies	Varies
$2a_p$, μ	Varies	Varies	Varies	21	9.5	3.5
ρ_p, g cm^{-3}	1.5‡	1.5‡	1.5‡	1.06	1.06	1.06
$\tilde{\epsilon}$	81	81	81	81	81	81
κ, cm^{-1}	2.8×10^6‡	2.8×10^6‡	2.8×10^6‡	2.5×10^6‡	2.5×10^6‡	2.5×10^6‡
ψ_{01}, mV	−30‡	−30‡	−30‡	−10‡	−10‡	−10‡
ψ_{02}, mV	−8‡	−8‡	−8‡	−5‡	−5‡	−5‡
$H \times 10^{13}$, erg	5*	5*	5*	1.01	1.01	1.01

‡ Estimated. * Estimated in Payatakes (1973).

For the estimation of the constriction size distribution of the beds used by Craft as well as other pertinent quantities, the procedure outlined in the Supplement was followed. The parameter values are given in Table 3 and the comparison between the theoretical and experimental values is shown in Figure 5. The theoretical model predicts correctly the experimentally observed effect of the suspended particle diameter on η_0 and is also in satisfactory quantitative agreement with the data, especially in view of the large experimental error bounds reported by Craft.* Theoretical calculations for suspended particle diameters larger than 30μ were not carried out, because for such particles the effect of the unit cell wall on the hydrodynamic force and torque acting on the suspended particles cannot be properly assessed based on the sphere-plate model, on which the trajectory equation [Equation (38), Part I], is based. This limitation of the utility of the trajectory equation is not due to a deficiency of the P-T-T porous media model, which should be valid for suspended particles of all sizes, but to the analytical difficulties involved in estimating the hydrodynamic force and torque setting on a relatively large sphere placed in a unit cell of the type used in this work. The comments made above concerning the nonsphericity of the particles used in the experiment hold in this case, too.

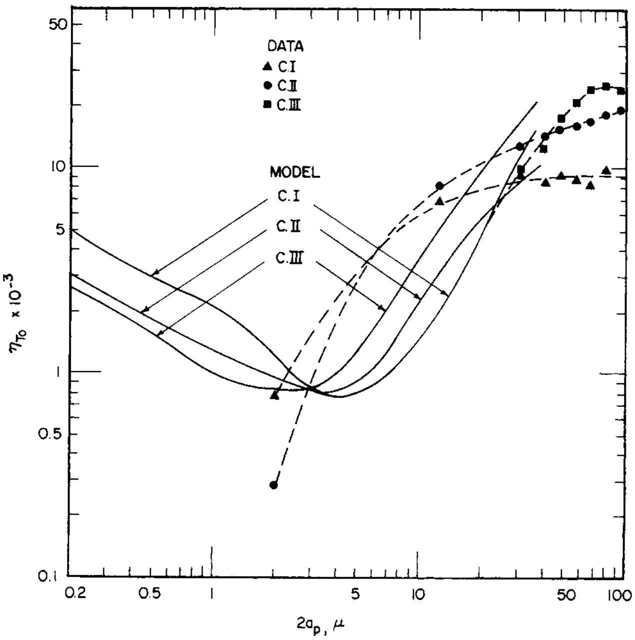

Fig. 5. Comparison between runs C.I, C.II, and C.III by Craft and the corresponding theoretical values.

Comparison with Data and Calculations by FitzPatrick (1972)

FitzPatrick obtained filtration data using suspensions of nearly monosized latex spheres in electrolyte solutions, and packed beds of glass spheres. As in previous cases, the bed characteristics in terms of the P-T-T model had to be estimated with the procedure outlined in the Supplement and the values are given in Table 3. The values for κ, ψ_{01}, and ψ_{02} reported by FitzPatrick differ from one experimental point to the other, and for this reason the values for these variables appearing in Table 3 are only approximately equal to the experimental ones. Since the experimental conditions of FitzPatrick's work were such that the electrokinetic force is negligible as compared to the molecular dispersion force at all separations, this discrepancy is unimportant. The experimental values of λ_0 are plotted in Figure 6 together with the corresponding values calculated with the FitzPatrick model and the model developed in the present work.

As can be seen, the theoretical values of FitzPatrick's are considerably higher than those of the present work.

* The data points corresponding to $12.5\mu = 2a_p$ are results reported by Craft for particles with size in the range 0-25μ. The arithmetic mean diameter is of questionable value in representing such a wide range of values. If the mass mean diameter were available, this would be more meaningful and would probably be closer to 20μ. This would result in better data correlation.

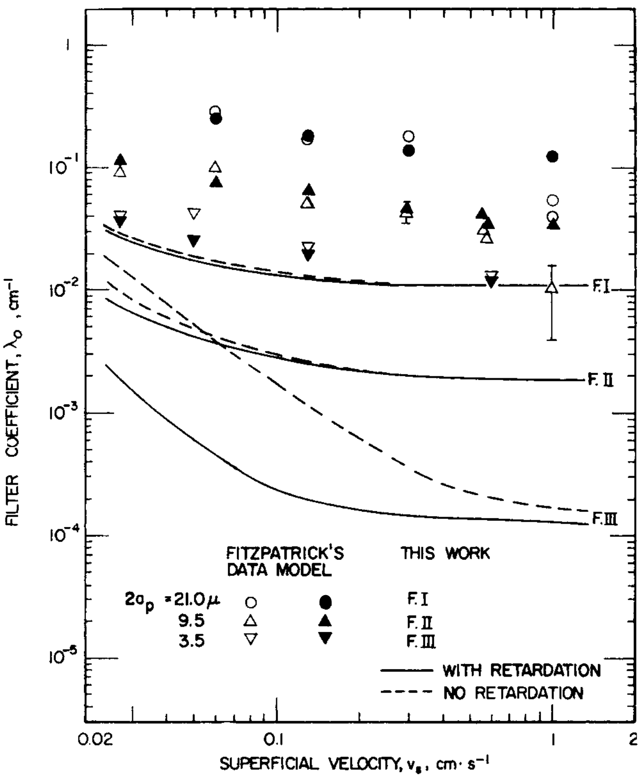

Fig. 6. Comparison between Fitzpatrick's data and the corresponding theoretical values.

It should be pointed out that the theoretical values of η_0 given by FitzPatrick are not strictly comparable to the theoretical values of this work, mainly because the former have been calculated without including the retardation effect. In order to demonstrate the fact that omission of the retardation effect could lead to significant overestimations of η_0 and λ_0, calculations were repeated neglecting the retardation effect, and the results are plotted in Figure 6 with dotted curves. The overestimation due to omission of the retardation effect becomes very large (more than 750%) for small particle diameters ($2a_p = 3.5\mu$) and small superficial velocity values ($v_s = 0.1$ cm/s) and diminishes for large particle diameters and large superficial velocity values (see also Supplement for additional results). Some unpublished results by FitzPatrick indicate that his model is less sensitive to the retardation effect, the overestimation under the aforementioned conditions being 300%, as compared to >750%.

A further comparison of the predictions of the theoretical model developed by FitzPatrick and the one developed in this work is given in Table 4 which also includes the experimental data obtained by Ison (1967). As can be seen, the agreement between the theoretical values obtained with the model of the present work and these experimental data is closer than that of the values based on FitzPatrick's model. It should be noted that the criterion developed by FitzPatrick (1972) predicts that for $2a_p = 9\mu$ the conditions used by Ison (1967) correspond to non-negligible double layer repulsion. This does not appear to be correct since the experimental values of the filtration coefficient are too high (very little deposition occurs in the case of non-negligible double layer repulsion), and also by the calculated values based on our model. The fact that FitzPatrick's criterion predicts otherwise for $2a_p = 9\mu$ could be attributed to its approximate nature and to the value of H he used (Table 4). The results reported by FitzPatrick for $2a_p = 9\mu$ are the calculated values based on his model.

TABLE 4. COMPARISON BETWEEN EXPERIMENTAL FILTER COEFFICIENT VALUES[1] OBTAINED BY ISON (1967) AND THE CORRESPONDING THEORETICAL VALUES BASED ON FITZPATRICK'S MODEL AND THE MODEL OF THIS WORK

Run	Experim. $\lambda_0 \times 10^2$	FitzPatrick[4] $\lambda_0 \times 10^2$	This work[2,3] $\lambda_0 \times 10^2$	This work[2,3] $\lambda_{T0} \times 10^2$
		$2a_p = 2.75\mu$		
I	6.0	7.0	1.3	2.4
II	8.1	8.6	1.9	3.1
III	11.0	10.1	2.8	4.1
IV	3.1	6.6	0.9	1.8
V	3.1	5.6	0.6	1.2
VI	4.5	5.9	1.4	2.1
VII	3.9	4.2	1.3	1.9
VIII	2.7	3.6	1.3	1.7
		$2a_p = 4.5\mu$		
I	7.6	13.7	2.8	3.2
II	11.0	17.5	3.7	4.6
III	15.0	21.3	5.3	6.2
IV	4.6	12.4	1.7	2.5
V	4.4	11.3	1.2	1.7
VI	5.8	9.6	2.9	3.3
VII	4.4	9.7	2.7	3.1
VIII	3.9	6.8	2.9	3.1
		$2a_p = 9\mu$		
I	8.8	37.6	6.7	7.1
II	14.0‡	41.0	9.6	10.5
III	16.5‡	52.7	13.8	14.2
IV	6.4	33.5	4.6	5.0
V	5.6	31.6	3.2	3.7
VI	6.3	30.0	7.3	7.7
VII	4.6	27.3	8.1	8.2
VIII	5.3	23.9	8.2	8.2

[1] The values of λ_0 are given in cm^{-1}.

[2] λ_{T0} is given by $\lambda_{T0} = \frac{1}{l} \ln \left(\frac{1}{1 - \eta_{T0}} \right)$.

[3] The parameter values used for the calculation of λ_0 and λ_{T0} are given in Table 2.

[4] FitzPatrick's calculations were based on the value $H = 10^{-13}$ erg for the Hamaker constant.

‡ These values are different from those quoted by FitzPatrick (1972), which are in error due to a typographical error. (Private communication with Dr. J. A. FitzPatrick).

ACKNOWLEDGMENT

This work was performed under Grant No. 3378452, New York State Science and Technology Foundation, and Grant No. GK-33976, National Science Foundation.

NOTATION

See Part I of this paper on page 897.

LITERATURE CITED

See Part I of this paper on page 899.

Manuscript received April 9, 1973; revision received January 14 and accepted April 19, 1974.